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APPLICATION OF SATELLITE GRADIOMETRY MEASUREMENTS TO LOCAL GEOI--ETC(U)
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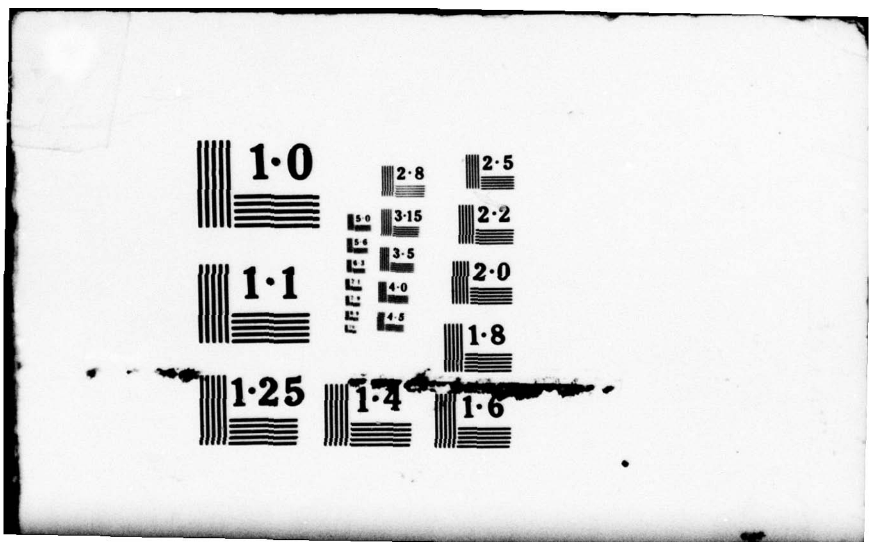
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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

APPLICATION OF SATELLITE
GRADIOMETRY MEASUREMENTS TO
LOCAL GEOID DETERMINATION

by

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FOREIGN TECHNOLOGY DIVISION

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Application of Satellite Gradiometry Measurements to Local Geoid Determination

J. Krynski, K. Schwarz *

1. Introduction

The problem of satellite gradiometry up to this time has been usually examined in a global aspect. With such an approach it is convenient to use interference potential, presented as a development into a series of spherical harmonics. Keeping in mind the decrease of the interaction of individual harmonics coefficients of the potential together with its absence from the Earth's surface and the sensitivity of the measurement device , measurements of a model of the Earth's field of gravitation can be gauged which can be obtained with a specific measuring accuracy. A designated model of a field of gravitation was taken to show the characteristic development into a series of spherical harmonics across the upper borders of the degree and order. In this case it is necessary to establish a global and uniform observational coverage.

Simulated tests conducted by Chovitz (2), (3) conclude that at a nominal measuring accuracy of $\pm$ 0.01 E (Eötvös) and a satellite altitude of 300 km the coefficients of the harmonics of the gravitational potential can be defined up to the degree and order of 70, with the limitation that serious complications will accompany the determination of a complete collection of harmonic coefficients.

The tests recently conducted by Rummel in which the author engages a good deal of attention to "downward continuation" in relation to satellite gradiometry lead to the conclusion that, based on the analysis of spectral correlation, it is possible to use satellite gradiometry in local aspect. Reed (11) devoted his work to the local use of satellite gradiometry. The author defined average anomalies for areas $2^{o} \times 2^{o}$ on the basis of simulated gradiometric tests. He showed that for this purpose it is sufficient to use limited observational material and that the results obtained accurately full the model of potential to the degree and order of 90.

The subject of this paper is the testing of the possibility of using satellite gradiometry to define the local geoid course. Taking advantage of the fact that much attention is given in the bibliography to the analysis of spectrum frequency of representatives of interference potential, this paper is limited to the testing of gravitational fields. From the viewpoint of practical applications it is of use to know that in local applications global covering by observations on satellite altitude is not required. Moreover, very modest observational material can be used. Besides this, there is no need to define the influence of individual gradients; their combined influence can be determined and the sum reaction of the combination of the heterogeneous data can be tested. Satellite gradiometry can be used not only for areas covered by land observations but also for areas completely devoid of them. In this instance the altitude of a geoid calculated from one of the

global solutions will be considered together with gradiometric measurements. In this way a geoid can be corrected locally until a satisfactory amount of information can be achieved to obtain a better global solution.

Instrumental problems were not a subject for discussion in this paper. A description of the instruments and their operating principles can be learned by studying numerous publications, e. g., (1), (4), (5), (16). So far, however, it is difficult to foretell what accuracy of gradiometric testing can be achieved from the SSZ layer. Hence, for the purpose of avoiding the drawing of too optimistic conclusions, various measuring accuracies are suggested in the paper.

The collocation method was shown to be unusually useful to solve the problem posed. The method makes it possible to estimate the expected accuracy on the basis of covariance function and the accuracy of observational data. Using collocation it is also possible in an elegant and efficient manner, to handle heterogeneous data together, such as: geoid undulation, gravimetric anomalies, and second order gradients.

2. Adjustment Process Plan

Assuming that observations are not laden with systematic errors a simple calculative model is used

where

$$s = C_{sx}\overline{C}^{-1}x, \qquad (1)$$

$$\overline{C} = C_{xx} + C_{nn}. \qquad (2)$$

The s vector, often called the "signal," contains elements which we plan to determine in the course of the calculations. In the case described they will be geoid undulations and vector s will have only one component. Vector x is the observation vector. Its components will be gradiometric measurements obtained from the satellite layer and on-ground observations of gravimetric anomalies and geoid undulation. The connection between the s signals and x observations is expressed by the covariance matrix C_{sx} and $\overline{C}$. To distinguish the covariance from among the various quantities the proper indexes are used. Thus through C_{xx} and C_{nn} the proper autocovariant matrix of the x observation is denoted and of measurement errors n, while C_{sx} denotes the covariation matrix between the s signal and x observations. Covariance matrixes contain all information on the structure of the gravitational field and on the accuracies of observation, hence the definition of the proper covariance function becomes an integral part of the task and as such, is subject to separate discussion.

The covariance matrix of errors E_{ss} represents the accuracy of designated altitudes

$$E_{ss} = C_{ss} - C_{sx}\overline{C}^{-1}C_{sx}^{T}. \qquad (3)$$

Using pattern (3) the accuracy of determination of a local geoid course was defined from a different measurement accuracy of satellite gradiometry. It should be noted that the results obtained in this manner have a fully overall pattern. They represent the limitless number of possible examples in practice contrary to the results of simulated testing, which characterize only the properties of singular examples. On the other hand, however, it proved the support of similar tests since with a proper selection of examples the results of simulated tests vary very insignificantly from the estimated accuracy using the method described.

3. Selection of Covariance Function

The covariance function, as already mentioned, includes information on the structure of the gravitational field. It must have the following properties:

-- representation of the statistical character of the gravitational field;

-- a simple analytical form guaranteeing simplicity in calculating derivatives;

-- isotropy and homogeneity (stability with respect to two linear transformations: rotation and displacement).

Moritz(9) proved that covariance functions can be described using three basic parameters. These parameters are related to the overall rotation-symetrical form and harmonic covariance function

$$K(P, Q) = A \sum_{l=0}^{\infty} k_l \left(\frac{R_B^2}{r_P r_Q} \right)^{l+1} P_l(\cos\psi), \tag{4}$$

where P and Q are two points in a space having geocentric radii r_P and r_Q, ψ is the angle between the radii r_P and r_Q, R_b is the right sphere radius usually called Bjerhammar's sphere, submerged completely in the middle in the terrestrial globe having the radius R = 6371 km, $P_l(\cos \psi)$ are the Legendre polynomials, while k_l designates certain coefficients having positive values. By defining k_l coefficients in a different way various models of covariance functions can be obtained.

Known to be fundamental in the composition of parameters of the covariance function C(s) are:

C_o -- variance

ξ -- correlated distance of covariance function

χ -- curvature parameter

The geometric interpretation of these parameters has been presented in Figure 1. The C_o variance is the value of the covariance function $C_{(s)}$ for the operand s = 0

$$C_0 = C(0). \tag{5}$$

The correlated distance ξ answers such a value for the operand of the covariance function C(s), for which there is the equation

$$C(\xi) = \frac{C_0}{2}. \tag{6}$$

The curvature parameter χ is a dimensionless quantity. It can be expressed as a function of the curvature of the $\varkappa$ covariance angle at the point s = 0 by the model:

$$\chi = \frac{\varkappa \xi^2}{C_0}. \tag{7}$$

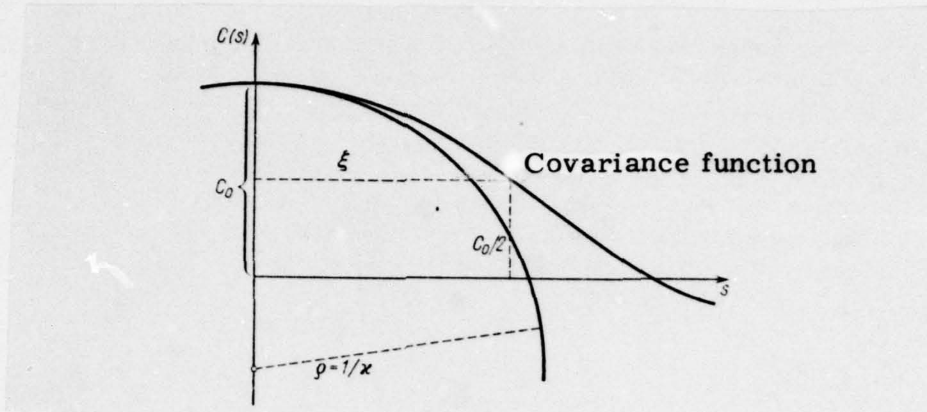


Fig. 1. Geometric interpretation of fundamental parameters of covariance function

The collection of parameters C_0, ξ and χ characterizes the properties of covariance functions for small and medium distances. This does not signify, however, that two covariance functions whose fundamental parameters are correspondingly equal are equivalent. A similarity in properties in relation to interpolation will calibrate such functions, i. e., errors in interpolation in both instances will be about the same. The C_0 variance defines scales of interpolation errors, the curvature parameter χ characterizes the properties of the covariance function in the case of small distances, but the correlated distance ξ is a measure of average distance and for such distances it describes the properties of covariance functions. Hence, this results in the fact that the shape of the covariance angle for the s operands exceeding the quantity 1.5 ξ does not play a more essential role, however the actual selection of the χ curvature parameter is very important since its size has considerable significance in the interpolation on small s distances. For this reason a description of the covariance function $C_{(s)}$ using only the two parameters C_0 and ξ is not satisfactory and can lead to erroneous results. It

appears that the variance G_O of the horizontal gradient of the gravimetric anomaly is more convenient for statistical estimation rather than the curvature parameter χ ~~X~~. Changes in the quantity G_O depending on the model chosen are very insignificant and do not exceed 3%. The quantity G_O, which is treated in this paper as the third basic parameter of the covariance function in place of the parameter of the χ ~~X~~ curvature, is expressed as a function of χ ~~X~~ in the following pattern:

$$G_0 = \frac{\chi C_0}{\xi^2}. \tag{8}$$

In practical applications it is usually accepted that C_O signifies variances of gravimetric anomaly.

The three basic parameters of the covariance function are defined statistically in support of the observational material. Accessible observational material is still far from ideal. It does however, allow for the elimination of erroneous assumptions in relation to the parameters mentioned.

As a rule, we assume that we dispose of the field of reference of the degree and order L determined from satellite observations, hence there is no need to execute a global estimate of the three basic parameters, Removing the influence of the first L coefficients in equation (4) we get a covariance function having a regional character in the form

$$K(P,Q) = A \sum_{l=L+1}^{\infty} k_l s^{l+1} P_l(\cos\psi), \tag{9}$$

where $s = \frac{R_B^2}{R^2}$ and $R = r_P = r_Q$.

This draws to attention the fact that the effect of subtracting a certain number of coefficients of a lower order from a representation of a covariance function is absolutely not equivalent to the acceptance of another type of covariance function. In the case of a geoid undulation, variance and correlated distance undergo considerable changes while for gradients of the second order the covariance function is nearly constant, but for the gravimetric anomaly very mild changes can be observed. As a result of a penetrating study of published material, the following numerical values of the parameters of the regional covariance function have been chosen:

$$C_0 = 1600 \text{ mgal}^2, \qquad \xi = 75 \text{ km}, \qquad G_0 = 300 \text{ E}^2.$$

An adjustment of the covariance function (9) to the above parameters requires a definition of the quantity of A, s, and L. Further on we can effect a selection of the k_l coefficient from among a series of known models. Among the parameters of both collections only C_0 and A are linearly dependent. The remaining parameters can generally be obtained solely by way of sequential approximations. One of the methods of moving one collection of parameters into another was described by Schwarz (14).

For the purpose of collecting a covariance function suitable for the problem posed a series of experiments on various models was conducted. The best model appeared to be the logarythmic type with a k_l coefficient, expressed by the following:

$$k_l = \frac{1}{(l-1)(l-2)(l+B)}. \qquad (10)$$

The numerical value of constant B appearing in pattern (10) was drawn form Tscherning and Rappa's report (18). It was not changed along with changes in the size of A and s, since in the case discussed strict adjustment to empirical variances of lower degrees does not play an essential role. Considerably more essential is a good approximation of the G_o parameter, whose size remains in a very strong connection with the sizes of gradients of the second order. Finally, the following numerical quantities have been accepted for A, s, and L:

$$A = 607.57 \text{ mgal}^2, \quad s = 0.998444, \quad L = 7,$$

where A agrees with the definition given by Tscherning and Rappa (18) and presents a constant value for the covariance function of a gravimetric anomaly. The accuracy of a statistical estimate of these quantities fall within the limits of units of suitable values.

Figure 2 shows three covariance functions founded on the chosen model for altitude h=0 km and h=300 km. The first of them, designated K (ψ) shows a covariance function of a geoid undulation, the second -- C (ψ) -- of gravimetric anomalies, while the third G (ψ) of the radial gradients of the second order. It must be noted that for every case a different scale for the ψ axis has been accepted and that the K (ψ) function differs from the one used in equation (9) with a constant coefficient.

The preliminary analysis of Figure 2 already leads to several interesting conclusions. The geoid undulation variance obtained $K(0) = 100m^2$ seems to be

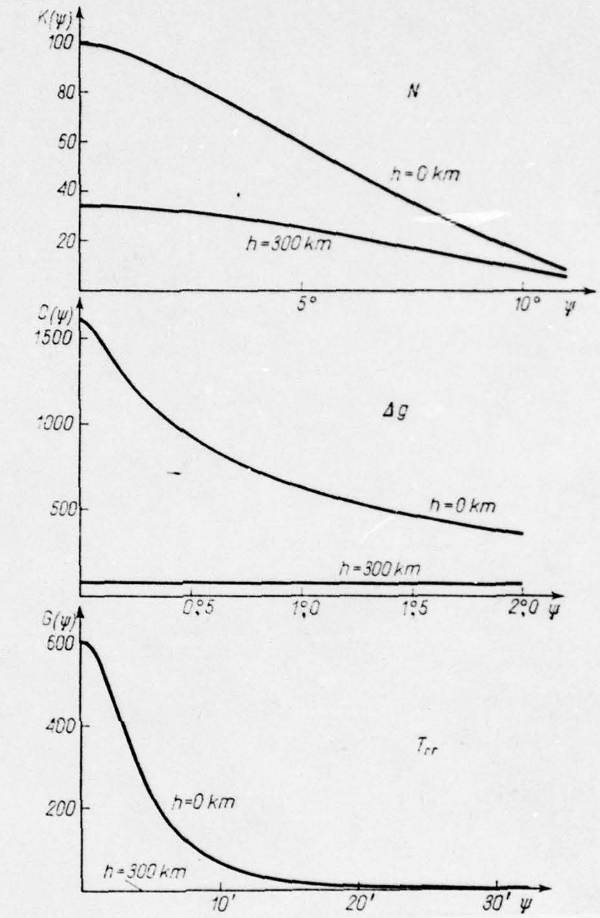


Fig. 2. Course of covariance function for geoid undulation K (ψ), gravimetric anomaly C (ψ) and gradient of the second order G (ψ)

relatively large. Its size, however, can be explained by the fact that a small number of variances of the lower orders have been removed. The reason for not using a global solution of a higher order for the reference model of the gravitational field is the fact that only the harmonic coefficients of the lower orders are not mutually

correlated. Hence, removing the influence of harmonics of lower orders which create reference fields, a significant correlation in the data is not moved in. The correlated distance ξ_0 undergoes a decrease during passage from geoid undulation through gravimetric anomalies to gradients of the second order indicating growth of localability in successive collections of data. On the other hand, attenuation of the angle pattern occurring with the altitude increase is strongest for gradiometric observations and in this case, the correlated distance undergoes the greatest changes along with the growth of h. In compliance with expectances the correlation between geoid undulations and gradients of the second order rises together with the altitude. This indicates that not only the absolute size of the signal plays a role, but also the mutual connection between the observations of various types have essential meaning.

4. Results

Test results will be published in a separate report by Krynski and Schwarz (6), thus discussion is limited only to several of the most important problems, namely:

1. Distribution of observations and selection of gradients,
2. Optimal satellite altitude,
3. Effect of measurement errors,
4. Combination of gradiometric observations with gravimetric anomalies.

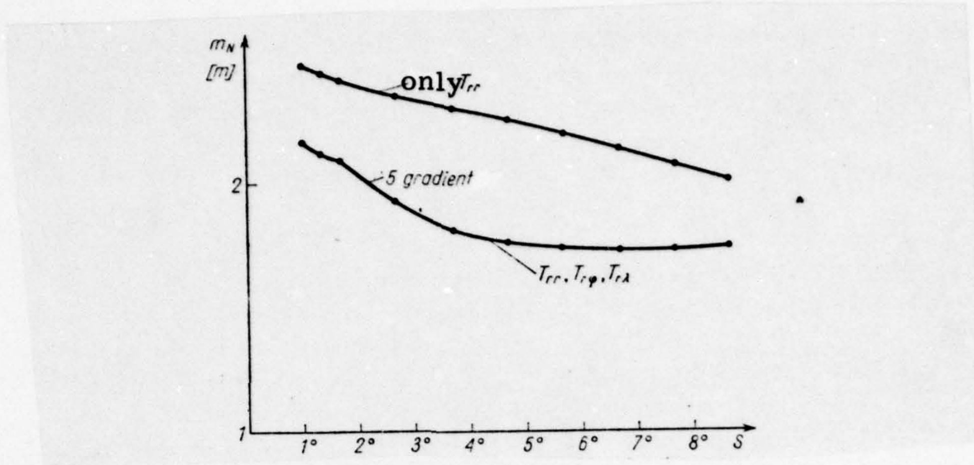
In local applications usually it suffices to be limited to observations which are inside a certain circle, in the center of which there is a designated point. Measurements on the exterior of this circle (critical circle) will have an insignificant effect on the estimated size. Actually the defined radius of a critical circle is nothing more than the correlated distance ξ characteristic for a given covariance function answering a given collection of observations. Accuracy in determining a given size depends on the distribution of observations and also on the quantity of measured gradients in the points lying on the interior of the critical circle. The size of the matrix, which is subject to inversion, is just proportional to the number of observations, thus before systematic analysis takes place finding an optimal configuration becomes necessary. In our case this task was formulated in the following way: In the system of co-ordinates (r, φ, λ) the point on the surface of the Earth is given ($h = 0$ km), in which the geoid undulation N is known from one of the global solutions with a mean error of ± 3 m. In the same system gradiometric observations are also given having accuracy of ± 0.01 E executed at an altitude of $h = 300$ km over a deliberate point. It is necessary to find such a configuration of observations and such a combination of gradients of the second order, which in an optimal way contribute to the adjustment of the accuracy of the N designation while keeping in mind a limited number of observations.

From the analysis of a series of the considered examples conclusions have been drawn which characterize the sought after optimal solutions. Gradiometric

observations should be distributed on a sphere symetrically as regards projection on it of a designated point with the co-ordinates φ, λ. They should be uniformly distributed in an area measuring about $5^{o} \times 5^{o}$, The distance between points should equal about 1^{0}. However, the question of distance between observations appears to be less essential than their uniform distribution in a $5^{o} \times 5^{o}$ area. It should be noted that the correlated distance of the covariance function depends on the altitude and for that reason area sizes as well as the distance between observations are also functions of the altitude.

Figure 3 illustrates results of the calculations on the basis of which selection of the proper gradients of the second order can be performed. From the combinations T_{rr}, $T_{r\varphi}$ and $T_{r\lambda}$ equally good results are obtained as from use of all five mutually independent gradients, however, only the gradients including a radial derivative influence the solution. Such a result could be foreseen from the discussion in Rummel's paper (12). A certain surprise, however, is the appearance of such a considerable difference between the best solution and the solution regarding T_{rr} solely, In the following calculations the optimal configuration of 17 points is used, in which T_{rr}, $T_{r\varphi}$, and $T_{r\lambda}$ are taken as observations.

The principle, which was accepted for applications with a global character, is that the orbit of a satellite should be reasonably near the Earth. Thus, reflecting upon the optimal altitude of a satellite, on which gradiometric observations are



Measurements of $5^{\circ} \times 5^{\circ}$ area, from which gradiometric data comes.

Figure 3. Accuracy to determine geoid undulation in support of observations of various gradients of the second order.

made may appear to be unimportant. From Figure 4a however, it appears that the problem of satellite altitude demands special deliberation from the viewpoint of local applications. Figure 4a presents the average error m_N in determining geoid undulation depending on the satellite altitude h. The average error reaches minimum at a height of about 400 km and accuracy lessens equally in the case of smaller as well as greater altitudes. Confirmation of the result obtained was achieved by using several other covariance functions. It appears that the optimal altitude is dependent upon the choice L, i.e., on the degree and order of the accepted reference field. This connection is exhibited in Figure 4b. A decrease of optimal altitude contributes to the growth of L. What is more, it decreases the mean error of the result together with a decrease of optimal altitude. Both phenomena equal out, however, when L grows while the altitude remains constant. This

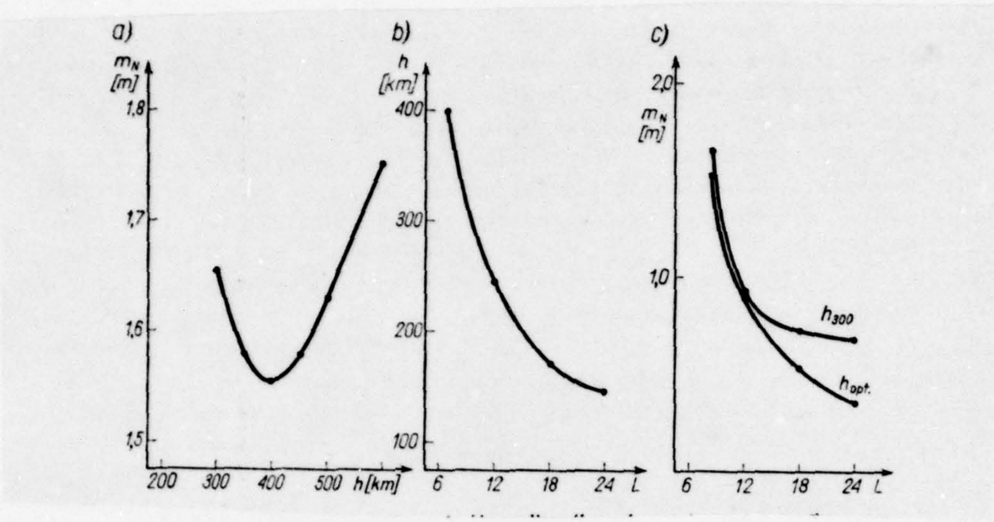


Figure 4a. Optimal satellite altitude for field of reference L=7.

Figure 4b. Dependence of optimal altitude on degree and order of field of reference.

Figure 4c. Accuracy of definition of geoid undulation from observation at an altitude of 300 km and at optimal altitude in function of degree and order of field of reference.

can be proven from Figure 4c, where the curve designated h_{opt} represents the mean error answering the optimal altitude, while the curve designated h_{300} represents the mean error answering the altitude h = 300 km for the field of reference of the L degree and order. The point of intersection of both curves defines the minimal L size, which should be used to elaborate the observations made at 300 km altitude. This is a field of the 12th degree and order, while the accuracy of geoid designation is estimated at less than one meter.

From the point of view of applications, the physical interpretation of these results is interesting. As it is known, the harmonic coefficients of the lower degrees and orders effect the most significant geoid deformation, and moreover, contribute to define the geoid undulation N. Using the field of reference of a low degree and order we reduce considerably the geoid variations. The geoid variations reduced in this manner have a characteristic equally global as well as local. These variations can be designated solely from measurements made in limited areas. Since effects having a global character have the greatest influence on the solution, it is preferable to use observations which represent the global effects well. This is connected however with the tolerancing of certain losses of information of local origin. Such a situation occurs when we consider the gradiometric measurements made in high orbits. Such measurements represent considerably better global effects than the local ones and for that reason give better results using reference fields of a low degree and order in comparison with such same observations made in lower orbits. The situation undergoes a change when it takes advantage of a field of reference of a higher degree and order. Then effects of a global character are represented through a field of reference and in such an instance in order to obtain a good solution regarding local influences observations should be used which are made in a low orbit.

Similar conclusions can be reached by analizing the covariance functions presented in Figure 2. Local gradiometric measurements at the altitude h= 0 km

have no effect on the global definition of the geoid undulation N. This is obvious if the differences in correlated distances of both covariance curves are weighed. Two values of ξ can come close together in a twofold manner. In the first instance the larger number of harmonic coefficients can be subtracted, through which variance and the correlated distance of the geoid undulation N undergo a considerable reduction. This, however, does not influence in a considerable fashion the covariance functions of gradients of the second order. In this way covariance functions for N are obtained which have an outstanding local character. In the second instance, on the other hand, we can remain with the original covariance function for geoid undulation instead of deliberating gradiometric observations at various altitudes. The correlated distance of the covariance function for gradients of the second order will rise together with the increase in altitude during the time when the variance will decrease very quickly. This effect is caused by smoothing the coefficients of the higher degrees and orders by the expression s^{2l+1} in formula (9). In this way high frequencies, which represent local qualities are entirely attenuated while low frequencies undergo only partial attenuation. This is reflected in the size of numerical variance on the satellite altitude, which size represents chiefly the effect of the harmonic coefficients of the lower orders. Together with a growth in size, the relative influence of low frequencies increases, which play a principal role in the determination of geoid undulation.

A problem of practical significance is the choice of a proper field of reference. For a proposed altitude of $h = 300$ km the optimal field of reference hovers within limits of from 12 to 15 in degree and order. Formula (3) can be used only then when all coefficients in the field of reference are satisfactorily well defined and when significant correlations do not appear among them. However, the models of gravitational field of the Earth known up to now from satellite observations do not make good this type of supposition. In connection with the above consideration of reciprocal correlations become necessary and for that reason in such instances other formulae (e.g., Rao (10)) should be used in the calculative process.

Another approach to the problem which makes it possible to avoid complications caused by mutual correlation of harmonic coefficients is to be limited only to that part of the model of the field in which the correlations are small through omission, or to harmonics of a lower order. In this case, however, we meet with a considerable increase of mean error. The above observation plays a particularly essential role in this discussion. Owing to it, it was possible to avoid a series of mistakes depending on the drawing of too optimistic conclusions.

In the calculative process the limited model of reference fe field making $L = 7$ was used. Supported by Schwarz's tests (13) it must be deemed that the establishment of mutual independences of harmonic coefficients in such a matched

model does not influence the results.

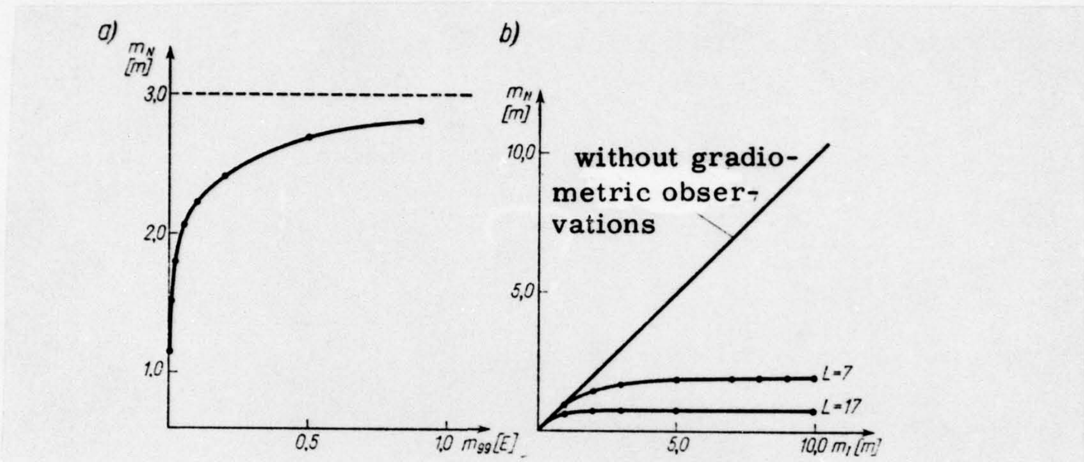


Figure 5a. Effect of mean errors of gradiometric observations on accuracy in defining geoid undulation.

Figure 5b. Role of gradiometric observations in adjusting determination of geoid undulation.

On the basis of Figure 5 we can make conclusions on the influence of measurement errors on the accuracy of defining the undulation of a geoid N. Calculations were made using the previously mentioned configuration of gradiometric observations executed at optimal altitude of $h = 400$ km, whose mean errors vacillate within limits of from $\pm$ 0.01 E to $\pm$ 1.0 E. The differences in the accuracy of designation N take on meaningfull sizes if the observation errors are less than a level of $\pm$ 0.05 E, while in cases of observations laden with errors exceeding $\pm$ 0.05 E a very slow increase of m_N can be noticed, which even then, when $m_{gg} = \pm 0.1$ E does not exceed a limit of $\pm$ 3 m. This phenomenon occurs also when output accuracy of the geoid undulation which is determined on the basis of one of

the global solutions is much worse from the previously accepted ± 3 m.

Figure 5a presents the influence of changes of output accuracy of the m_I geoid undulation, which wavers in a boundary of 0 m to ± 10 m, on the accuracy of the N determination when gradiometric observations are used. The curves appearing in the figure answer two different fields of reference: $L=7$ and $L=17$ and the satellite altitudes $h=300$ km. As is easy to notice, the mean error m_N undergoes essential changes only in instances of very small sizes of m_I. For $M_I > 3$ m the accuracy of determining the geoid undulation can be taken as a constant size. Consideration of gradiometric observations during geoid testing gives rather more calculable results when we do not dispose of a good model of gravitational field. Then even gradiometric observations having mean errors which exceed ± 0.05 E contribute in an essential way to the adjustment of results.

Considerations have been made to this time on the supposition that on-land observations are inaccessible. Thus, conclusions obtained which concern expected accuracy of geoid determination concern areas not covered by on-land observations. In cases where gravimetric observations were made in a tested area, these observations can be jointly elaborated with satellite observations. This establishes excellent possibilities of controlling systematic influences of each observation from the groups. The fundamental feature of the combined solution is the clear stabilizing of the frequency range. Accuracy adjustments should also be expected.

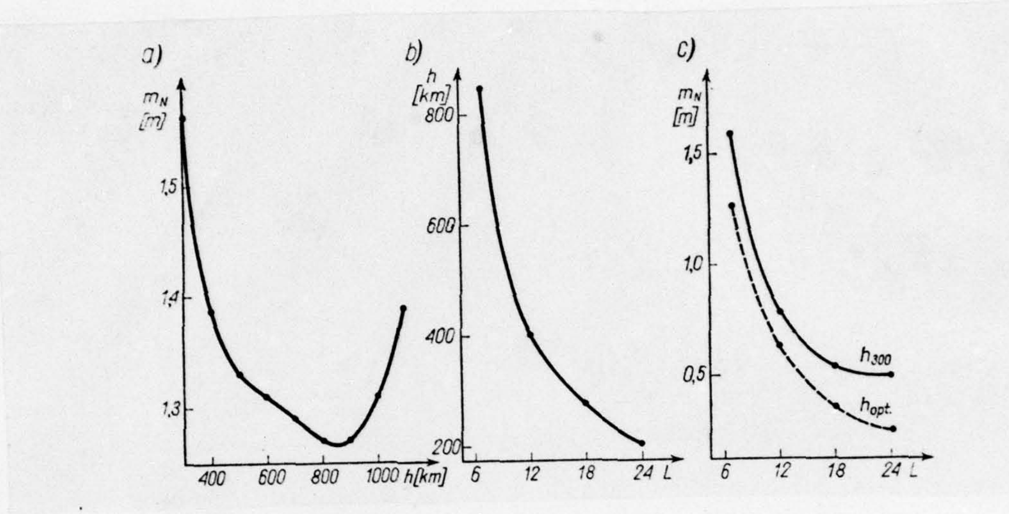


Figure 6a. Optimal satellite altitude for field of reference $L = 7$ (role of gravimetric observations)

Figure 6b. Dependence of optimal altitude on degree and order of field of reference (role of gravimetric observations)

Figure 6c. Accuracy in defining geoid undulation from observations at 300 km altitude and at optimal altitude in function of degree and order of field of reference (role of gravimetric observations)

See page 22a for omission.

Figures 6a, 6b, 6c answer the Figures 4a, 4b, 4c after adding gravimetric anomalies. Figure 6a discloses that the optimal altitude does not differentiate itself as clearly as before. It is within limits of from 450 km to 1100 km, in which the accuracy hardly differs from the accuracy for $h_{opt} = 850$ km. This means that if we dispose of several gravimetric observations in the tested area and we use a field of reference of low degree and order, then we can use the gradiometric observations made in a sufficiently high orbit. For fields of reference of higher degrees and orders the range of fixed accuracy is not as wide, nonetheless the

Omitted from page 22.

To the previously used model of gravimetric observations was added a collection of gravimetric observations symmetrically distributed with respect to a designated point and located within a radius of 0°, 4 around the designated point. The gravimetric observation errors m_G were assumed to be ±3 mgal. It was demonstrated that adding a greater number of gravimetric observations and changing their distance from the designated point do not significantly change accuracy in determining geoid undulation.

fundamental character of the conclusions remains without change. It can be noticed that the optimal altitudes on Figures 6b and 4b draw near each other with the increase of L.

Figure 6c shows that a gain in accuracy as is reached by joining the calculations of gravimetric observations wavers between 20% and 30%. By adding gravimetric observations better accuracies are achieved and also the optimal satellite altitude increases. If satellite altitude was established at h = 300 km, then even 60% accuracy adjustment (at L = 24) is reached. Hence the result that by elaborating jointly gravimetric and gradiometric observations the surface and reference can be determined for an accurate satellite altimeter.

5. Conclusions

Using gradiometric measurements and the collocation method the determination of a local geoid can be effectively adjusted. The following conclusions result from the analysis conducted:

1. Consideration of the observational material distributed in an area $5^{\circ} \times 5^{\circ}$ at a satellite altitude of h = 300 km is satisfactory.

2. The essential influence on the solution has only three gradients of the second order T_{rr}, $T_{r\varphi}$, and $T_{r\lambda}$.

3. For each field of reference having a definite degree and order there exists an optimal altitude at which satellite gradiometric observations should be made. For a field of reference of low degree and order this altitude considerably exceeds 300 km.

4. For gradimetric measurements at 300 km altitude of degrees and orders the fields of reference should waver within limits of from 12 to 15. When the harmonic coefficients are not correlated, then the accuracy in determining the geoid will be of the order of 1 meter.

5. For an essential adjustment of local geoid accuracy of observations better than ± 0.05 E is required. If, however, an error of global solution of a geoid exceeds ± 3 m, then a clear adjustment of the determination of the course of the geoid can be achieved also from less accurate gradiometric observations.

6. The combination of satellite gradiometry with on-land gravimetric measurements made in limited areas contributes to significant accuracy of the results. Then the optimal altitude of the satellite increases and the accuracy of determination of the geoid undulation raises as well. Adjustment of the accuracy can in this case reach even 60% at a satellite altitude of h = 300 km. Moreover, the combined elaboration of gradiometric and gravimetric measurements creates excellent possibilities for the control of systematic influences of each observation from the groups.

The above conclusions theoretically depend on the choice of covariance function. If, however, the ranges, in which the output parameters which are obtained from the contemporary statistical estimates, are real, then the differences to which the use of various covariance functions leads, are very small.

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